

Embedding Dark Energy in Supergravity

Philippe Brax

Service de Physique Théorique, CEA/DSM/SPhT, Unité de recherche associée au CNRS, CEA-Saclay 91191 Gif/Yvette cedex, France

Dark energy[1] is one of the most intriguing puzzles of present day physics. When interpreted within the realm of General Relativity, its existence is linked to the presence of a weakly interacting fluid with a negative equation of state and a dominant energy density. The simplest possibility is of course a pure cosmological constant. A plausible alternative involves the presence of a scalar field responsible for the tiny vacuum energy scale[2, 3, 4]. In most cases, the quintessence field has a runaway potentials and takes large values now, of the order of the Planck mass. This suggests to embed such models in high energy physics[5, 6]. The most natural possibility is supergravity as it involves both supersymmetry and gravitational effects. Moreover, superstring theories lead to supergravity models at low energy.

From the model building point of view, the quintessence field does not belong to the standard model. Hence there must be a separate dark energy sector. The observable sector is well-known and the hidden supersymmetry (SUSY) breaking sector can be parameterised[7]. In the following, we give a brief overview of some of the constraints on the embedding of dark energy in broken supergravity following mostly [8, 9, 10].

1 Coupling Dark Energy to SUSY Breaking

As soon as a quintessence field has a runaway potential and leads to the present day acceleration of the universe expansion, its mass is tiny and may lead to gravitational problems. In order to minimise this problem, we assume that the quintessence sector is only coupled gravitationally to the observable and hidden sectors. This can be described by the Kähler and super potentials

$$K = K_{\text{quint}} + K_{\text{hid}} + K_{\text{obs}}, \quad W = W_{\text{quint}} + W_{\text{hid}} + W_{\text{obs}}. \quad (1)$$

The observable sector comprises the fields of the Minimal Standard Supersymmetric Model (MSSM) ϕ^a and the corresponding superpotential can be expressed as

$$W_{\text{obs}} = \frac{1}{2}\mu_{ab}\phi^a\phi^b + \frac{1}{3}\lambda_{abc}\phi^a\phi^b\phi^c, \quad (2)$$

where μ_{ab} is a supersymmetric mass matrix and λ_{abc} the Yukawa couplings.

SUSY breaking causes the appearance of soft terms in the observable and dark sectors. We can parameterise the hidden sector supersymmetry breaking in a model independent way

$$\kappa^{1/2} < z_i >_{\min} \sim a_i(Q), \quad \kappa < W_{\text{hid}} >_{\min} \sim M_s(Q), \quad \kappa^{1/2} < \frac{\partial W_{\text{hid}}}{\partial z_i} >_{\min} \sim c_i(Q) M_s(Q), \quad (3)$$

where a_i and c_i are coefficients of order one which depend on the detailed structure of the hidden sector, M_s is the SUSY breaking scale and $\kappa \equiv 8\pi/m_{\text{pl}}^2$. Notice that the coupling of the hidden sector to quintessence implies that the vev's of the hidden sector fields z_i responsible for supersymmetry breaking can depend on the quintessence field. The observable potential reads

$$\begin{aligned} V_{\text{mSUGRA}} = & \dots + e^{\kappa K} V_{\text{susy}} + e^{\kappa K} \mathcal{A}(Q) \lambda_{abc} (\phi_a \phi_b \phi_c + \phi_a^\dagger \phi_b^\dagger \phi_c^\dagger) + e^{\kappa K} B(Q) \mu_{ab} (\phi_a \phi_b + \phi_a^\dagger \phi_b^\dagger) \\ & + m_{a\bar{b}}^2 \phi_a \phi_b^\dagger. \end{aligned} \quad (4)$$

where the soft terms are the terms which are not in V_{susy} .

We consider that the dark energy superpotential is of the form

$$W_{\text{quint}}(Q) \equiv M^3 \mathcal{W}(\kappa^{1/2} Q). \quad (5)$$

where M is a scale characterising dark energy. The choice of the Kähler potential is also crucial. As an example, we will focus on the no-scale case corresponding to Kähler moduli

$$K_{\text{quint}} = -\frac{3}{\kappa} \ln [\kappa^{1/2} (Q + Q^\dagger)], \quad (6)$$

The kinetic terms of the moduli read $3|\partial Q|^2 / (Q + Q^\dagger)^2$ implying that Q is not a normalized field. The normalized field q is given by

$$\kappa^{1/2} Q = \exp \left(-\sqrt{\frac{2}{3}} q \right). \quad (7)$$

where q is a dimensionless scalar field.

In the no scale case and if W_{hid} is constant, M_s is constant, \mathcal{A} and B are constant of the order of M_s , and

$$2B = -M_s + 3\mathcal{A}, \quad (8)$$

while the mass $m_{a\bar{b}}$ acquires a very simple Q -dependence given by

$$m_{a\bar{b}} = \frac{M_s}{[\kappa^{1/2} (Q + Q^\dagger)]^{3/2}} \delta_{a\bar{b}}. \quad (9)$$

In general, the soft terms have a non-trivial dependence on Q .

We now consider the application of the previous results to the electroweak symmetry breaking. The Higgs potential also becomes a Q -dependent quantity. The total Higgs potential, taking H_u^0 and H_d^0 to be real reads

$$V^{\text{Higgs}} = e^{\kappa K_{\text{quint}}} \left[(|\mu|^2 + m_{H_u}^2) |H_u^0|^2 + (|\mu|^2 + m_{H_d}^2) |H_d^0|^2 - 2\mu B(Q) |H_u^0| |H_d^0| \right] + \frac{1}{8} (g^2 + g'^2) \left(|H_u^0|^2 - |H_d^0|^2 \right)^2. \quad (10)$$

In presence of dark energy, the minimum becomes Q -dependent and the particles of the standard model acquire a Q -dependent mass. Straightforward calculations give

$$e^{\kappa K_{\text{quint}}} (|\mu|^2 + m_{H_u}^2) = \mu B(Q) \frac{e^{\kappa K_{\text{quint}}}}{\tan \beta} + \frac{m_{Z^0}^2}{2} \cos(2\beta), \quad (11)$$

$$e^{\kappa K_{\text{quint}}} (|\mu|^2 + m_{H_d}^2) = \mu B(Q) e^{\kappa K_{\text{quint}}} \tan \beta - \frac{m_{Z^0}^2}{2} \cos(2\beta), \quad (12)$$

where we have defined the Higgs vevs as $\langle H_u^0 \rangle \equiv v_u$, $\langle H_d^0 \rangle \equiv v_d$, $\tan \beta \equiv v_u/v_d$, and m_{Z^0} as the gauge boson Z^0 .

From the equations (11) and (12), one can also deduce how the scale $v \equiv \sqrt{v_u^2 + v_d^2}$ depends on the quintessence field. This leads to

$$v(Q) = \frac{2e^{\kappa K_{\text{quint}}/2}}{\sqrt{g^2 + g'^2}} \sqrt{|\mu|^2 + m_{H_u}^2} + \mathcal{O}\left(\frac{1}{\tan \beta}\right). \quad (13)$$

in the large $\tan \beta$ regime.

Then, finally, one has for the vev's of the two Higgs fields

$$v_u(Q) = \frac{v(Q) \tan \beta(Q)}{\sqrt{1 + \tan^2 \beta(Q)}} = v(Q) + \mathcal{O}\left(\frac{1}{\tan^2 \beta}\right), \quad (14)$$

$$v_d(Q) = \frac{v(Q)}{\sqrt{1 + \tan^2 \beta(Q)}} = \frac{v(Q)}{\tan \beta} + \mathcal{O}\left(\frac{1}{\tan^2 \beta}\right), \quad (15)$$

at leading order in $1/\tan^2 \beta$. This allows us to deduce the two kinds of fermion masses, depending on whether the fermions couple to H_u or H_d

$$m_{u,a}^F(Q) = \lambda_{u,a}^F e^{\kappa K_{\text{quint}}/2} v_u(Q), \quad m_{d,a}^F(Q) = \lambda_{d,a}^F e^{\kappa K_{\text{quint}}/2} v_d(Q), \quad (16)$$

where $\lambda_{u,a}^F$ and $\lambda_{d,a}^F$ are the Yukawa coupling of the particle ϕ_a coupling either to H_u or H_d . The masses pick up a $\exp(\kappa K_{\text{quint}}/2)$ dependence from the expression of $v(Q)$ and another factor $\exp(\kappa K_{\text{quint}}/2)$ from the definition of the mass itself. As a result we have $m \propto \exp(\kappa K_{\text{quint}})$. In no scale quintessence the behaviour of the standard model particle masses is universal and given by $m(Q) \propto \frac{1}{[\kappa^{1/2}(Q+Q^\dagger)]^3} \propto e^{-\sqrt{6}q}$.

After electro-weak symmetry breaking, the low energy action in the Einstein frame reads

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} - \frac{1}{2} g^{\mu\nu} (\partial_\mu q)(\partial_\nu q) - V_{\text{DE}}(q) \right) + S_m(\psi_u, A_u^2(q) g_{\mu\nu}) + S_m(\psi_d, A_d^2(q) g_{\mu\nu}) \quad (17)$$

where $K_{Q\bar{Q}}(\partial Q)^2 = \frac{1}{2}(\partial q)^2$ and $A_{u,d}(q) = \frac{m_{u,d}(q)}{m_{u,d}}$ is the ratio of the q dependent masses to their values in the absence of coupling to dark energy. Notice that in general, the particles $\psi_{u,d}$ coupling to $H_{u,d}$ do not couple to gravity in an universal way, hence a violation of the weak equivalence principle. In the following, we will neglect the q dependence of $m_{H_{u,d}}$ and B leading to $A_{u,d}(q) = A(q)$. This is exact in the no scale case.

If the dark energy potential is of the runaway type then this implies that the quintessence field has a mass $m_q \sim H_0$, *i.e.* of the order of the Hubble rate now. The range of the force mediated by the quintessence field is large. In order to satisfy the constraints coming from fifth force experiments such as the recent Cassini spacecraft experiment, one must require that the Eddington (post-Newtonian) parameter $|\gamma - 1| \leq 5 \times 10^{-5}$. If one defines the parameter $\alpha_{u,d}$ by

$$\alpha_{u,d}(Q) \equiv \left| \frac{d \ln m_{u,d}^F(q)}{dq} \right|, \quad (18)$$

where the derivative is taken with respect to the normalized field q , then one must impose that $\alpha_{u,d}^2 \leq 10^{-5}$ since one has $\gamma = 1 + 2\alpha_{u,d}^2$ [11, 12]. This leads to a bound on $\alpha_{u,d}(q) \approx \frac{1}{2} \partial_q K_{\text{quint}}$

$$\partial_q K_{\text{quint}} \leq 10^{-2} \quad (19)$$

Notice the analogy with the η problem of inflation. This constraint can be satisfied using an appropriate shift symmetry. In the no scale case, Eq. (1) implies $\alpha_{u,d} = \sqrt{6}$ in contradiction with the bounds on the existence of a fifth force.

However, the above description is too naive because we have not taken into account the chameleon effect. Indeed, in the presence of surrounding matter like the atmosphere or the inter-planetary vacuum, the effective potential for the quintessence field is modified by matter and becomes [13, 14]

$$V_{\text{eff}}(Q) = V_{\text{DE}}(Q) + A(Q) \rho_{\text{mat}}, \quad (20)$$

where $A(Q)$ is the coupling of the quintessence field to matter. This can lead to an effective minimum for the potential even though the Dark Energy potential is runaway. The theory is compatible with gravity tests if [13]

$$\frac{\alpha_q q_{\text{now}}}{\Phi_N} \ll 1. \quad (21)$$

Even if α_q is quite large, if the new factor q_{now}/Φ_N is small then the model can be compatible with gravity. This is the thin shell effect. It strongly depends on the shape of the potential and, therefore, on the Kähler and superpotential in the dark energy sector.

2 Quintessential Puzzles

Radiative corrections can modify the form of the quintessence potential. In the Jordan frame where standard model matter couples to $\tilde{g}_{\mu\nu} = A(q)g_{\mu\nu}$, the quintessence field only appears in the gravity part of the Lagrangian, i.e. the Newton constant becomes q -dependent. Now, integrating out all the standard model fields to obtain the effective action leads to the appearance of a cosmological constant term Λ_0^4 . No contribution involving q can appear as gravitational loops are not taken into account. Going back to the Einstein frame implies that the dark energy potential is modified by $\delta V_{\text{DE}} = \Lambda_0^4 A^4(q)$. The same result can be obtained using the covariance of the action in the Einstein frame. Of course, such a correction is huge as $A(q) = 1 + \dots$ [15]. This is the usual cosmological constant problem. Consistency imposes that Λ_0 must be very small. In the following, we implicitly assume that an unknown mechanism guarantees that $\Lambda_0 = 0$.

Let us come back to the structure of the scalar potential when the quintessence superpotential is small compared to the hidden sector superpotential $M \ll M_s$

$$V = V_{\text{DE}}(Q) + \sum_i |F_{z_i}|^2 + e^{\kappa K} (K^{Q\bar{Q}} K_Q K_{\bar{Q}} - \frac{3}{\kappa}) \kappa^2 |W|^2 \quad (22)$$

The first term V_{DE} contains terms of order M^4 and $M_s^2 M^2$, it is responsible for the quintessence property of the model. The second term contains the F-terms of the hidden sector. The third term lead to a potential for the quintessence field (if it does not vanish).

Let us consider first models where the Kahler potential can be expanded around $Q = 0$

$$K = Q\bar{Q} + \dots \quad (23)$$

where \dots represent Planck suppressed operators. The quintessence field picks up a soft breaking mass [16, 10]

$$V = V_{\text{DE}} + m_{3/2}^2 |Q|^2 \quad (24)$$

where we must impose $\sum_i |F_{z_i}|^2 = 3m_{3/2}^2 \kappa^{-1}$ in order to cancel the intolerably large contribution to the cosmological constant coming from the hidden sector. Due to the large value of $m_{3/2}$ compared to the quintessence field, the potential acquires a minimum Q_0 small in Planck units. The scale M is tuned to get a minimum value for the potential of order $\Omega_\Lambda \rho_c$. At this minimum, the mass of the quintessence field

is $m_{3/2}$, large enough to evade all the gravitational tests. Now cosmologically, the steepness of the quadratic potential in Q implies that the field must have settled at the minimum before BBN. If not the energy density of the quintessence field would exceed the MeV energy scale of BBN. In practice, the potential is constant since BBN, i.e. equivalent to a cosmological constant: a very intricate manner of modelling a pure cosmological constant throughout most of the universe history!

One can circumvent this argument by taking singular potentials where the potential term in $|W|^2$ is constant. One can choose

$$K = -\frac{n}{\kappa} \ln \kappa^{1/2} (Q + \overline{Q}) \quad (25)$$

In this case, $n=3$ for moduli and $n=1$ for the dilaton. Fine-tuning of the cosmological constant requires

$$\sum_i |F_{z_i}|^2 = (3 - n) m_{3/2}^2 \kappa^{-1} \quad (26)$$

leaving

$$V = V_{\text{DE}} \quad (27)$$

No mass term appears for the quintessence field. The mass of the quintessence field at the minimum of the matter-dependent potential is of order H_0 . Moreover the thin-shell effect is only present for small values of the normalised scalar field q . This is not the case for well-motivated superpotentials motivated such as the ones obtained from gaugino condensation. However, this is not excluded for clever choices of the dark energy superpotential.

In conclusion, coupling dark energy to supersymmetry breaking modifies runaway potentials in a drastic way, giving a large mass to the quintessence field of order of the gravitino mass. This can only be avoided using no scale models. In this case, only very special superpotentials can lead to a chameleon effect, and therefore viable models. The construction of such models is challenging and worth pursuing.

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References

- [1] S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999); P. M. Garnavich *et al.*, *Astrophys. J.* **493**, L53 (1998); A. G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998); P. Astier *et al.*, *Astron. Astrophys.* **447**, 31 (2006).
- [2] B. Ratra and P. J. E. Peebles, *Phys. Rev. D* **37**, 3406 (1998).
- [3] P. G. Ferreira and M. Joyce, *Phys. Rev. D* **58**, 023503 (1998).
- [4] E. Copeland, M. Sami and S. Tsujikawa, *Int. J. Mod. Phys. D* **15** (2006) 1753

- [5] P. Binétruy, Phys. Rev. D **60**, 063502 (1998), [hep-ph/9810553](#); P. Binétruy, Int. J. Theor. Phys. **39**, 1859 (2000).
- [6] P. Brax and J. Martin, Phys. Lett. **B468**, 40 (1999).
- [7] H. P. Nilles, Phys. Rept. **101**, 1 (1984); S. P. Martin, [hep-ph/9709356](#); I. J. R. Aitchison, *Supersymmetry and the MSSM: An Elementary Introduction*, Notes of Lectures for Graduate Students in Particle Physics, Oxford, 1004 (2005).
- [8] Ph. Brax and J. Martin, Phys. Lett. **B647** 320 (2007).
- [9] Ph. Brax and J. Martin, JCAP 0611:008 (2006).
- [10] Ph. Brax and J. Martin, Phys. Rev. D **75** 083507 (2007).
- [11] C. M. Will, Living. Rev. Rel. **9**, 2 (2006); E. Fischbach and C. Talmadge, *The Search for non-Newtonian Gravity*, Springer-Verlag, New-York, (1999); B. Bertotti, L. Iess and P. Tortora, Nature **425**, 374 (2003); G. Esposito-Farese.
- [12] T. Damour and A. M. Polyakov, Nucl. Phys. **B423**, 532 (1994).
- [13] J. Khoury and A. Weltman, Phys. Rev. D **69** (2004) 044026
- [14] P. Brax, C. van de Bruck, A. C. Davis, J. Khoury and A. Weltman, Phys. Rev. D **70**, 123518 (2004).
- [15] M. Pietroni, Phys. Rev. D **72** (2005) 043535.
- [16] S. Carroll and D. Lyth, Phys. Lett. **bf B458** (1999) 197.